Proof Without Words: The Golden Ratio

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**Theorem.** If \( x > 0 \) and \( x = 1 + 1/x \), then \( x = \varphi = (1 + \sqrt{5})/2 \).

**Proof.**

\[
(2x - 1)^2 = 5 \implies x = \varphi = \frac{1 + \sqrt{5}}{2}.
\]

**Exercise.** Show that \( \varphi^2 + (1/\varphi)^2 = 3 \). (Hint: In the figure, create a square with area 3 by drawing a diagonal in each of the four rectangles.)

**Summary.** We employ a square with area 5 to determine the golden ratio (without using the quadratic formula).

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