Proof Without Words: Each But Two Triangular Numbers Is a Sum of Three Triangular Numbers

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**Proposition.** Each triangular number \( T_n = \sum_{k=1}^{n} k \) except \( n = 1, 3 \) is the sum of three triangular numbers.

*Proof:* \( T_{3n-1} = 2T_{2n-1} + T_n \) for \( n \geq 1 \).

\[
T_{3n} = 2T_{2n} + T_{n-1} \quad \text{for} \quad n \geq 2.
\]

\[
T_{3n+1} = T_{2n+1} + T_{2n} + T_n \quad \text{for} \quad n \geq 1.
\]

**Summary.** We show wordlessly that each triangular number except 1 and 6 is a sum of three triangular numbers.

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